More applications of exponential equations

Simple Interest – interest is "paid out" annually (one time per year)

$\$ = P(1+r)^t$ $\quad$ $P = principal, \; r = annual\ interest\ rate, \; t = time\ in\ years$

Compound Interest – interest is "paid out" at multiple times per year

You get interest on your interest.

$\$ = P\left(1+\frac{r}{k}\right)^{kt}$ $\quad$ $k = number\ of\ times\ your\ money\ is\ compounded\ each\ year$

Note: $\frac{r}{k} = the\ fraction\ of\ interest\ you\ get\ at\ each\ time\ of\ compounding$

Interest Compounded Continuously – interest is "paid out" every time you blink.

$\$ = P \cdot e^{rt}$

Suppose you invest \$300 in the bank. The bank offers a 5% annual interest rate.

How much money will you have in 10 years, if the money is:

A. Compounded annually?

$\$ = P(1+r)^t = 300(1+0.05)^{10} = 300(1.05)^{10} = \$488.67$

B. Compounded monthly?

$\$ = P(1+\frac{r}{k})^{kt} = 300(1+\frac{0.05}{12})^{12\cdot 10} = 300(1.004)^{120} = \$494.10$

C. Compounded continuously?

$\$ = P \cdot e^{rt} = 300 \cdot e^{0.05\cdot 10} = \$494.62$

$500 = 300(1+\frac{0.05}{4})^{4t} \quad \rightarrow \quad 500 = 300(1.0125)^{4t} \quad \rightarrow \quad 1.67 = 1.0125^{4t}$

Convert: $4t = log_{1.0125}1.67 = \frac{log1.67}{log1.0125} = 41.28 \quad \rightarrow \quad t = \frac{41.28}{4} = 10.32\ years$

If the money is compounded daily, how long will it take to turn your investment into \$500?

$500 = 300(1+\frac{0.05}{365})^{365t} \quad \rightarrow \quad 500 = 300(1.000137)^{365t} \quad \rightarrow \quad 1.67 = 1.000137^{4t}$

Convert: $365t = log_{1.000137}1.67 = \frac{log1.67}{log1.000137} = 3743.87 \quad \rightarrow \quad t = \frac{3743.87}{365} = 10.26\ years$

Percent increase revisited:

The population of Gophertown is 40000 in 2000. The population increases 12% each year.
An alternate model is: $P = Ie^{bt}$
What is the population in 2013?

$In\ one\ year\ (t=1), the\ population\ will\ increase\ by\ 4800 = 0.12 \times 40000, \; so$

$44800 = 40000e^b \rightarrow \quad 1.12 = e^b \rightarrow \quad b = \ln 1.12 = 0.1133$

$P = Ie^{rt} = 40000e^{0.1133t} = 40000e^{0.1133(13)} = 174474$